Recall that losses typically depend upon two random variables: the number of losses in a specified period (called the *frequency of loss* and modeled with a *frequency distribution*) and the amount of the loss (called the *severity* and modeled with a *severity distribution*). Combining these two random variables will give us the overall loss distribution.

Example: Consider a homeowner who has an 85% chance of no claims in a year, a 15% chance of having a single claim in a year, and no chance of having more than one claim in a year. Assume that there is a 40% probability that the claim will require repairs costing $1500, a 50% probability that the claim will require repairs costing $6000, and a 10% probability that the house will need to be replaced, which will cost $150000.

1. Find the loss distribution $X$.

2. Find the homeowner’s expected loss.

3. Find the variability in the loss distribution by computing its variance and standard deviation.
Example: Consider an insurance company that will cover 100 homeowners, each with the same risk as in the previous example. Assume independence among the policyholders.

1. Find the expected loss of the group of 100 homeowners.
2. Find the variance and standard deviation of the loss random variable for the insurance company.
3. What net premium should the insurer collect, if the policy covers losses in full?
4. Why would the homeowner be willing to pay more than the net premium for this insurance?

Example: Now suppose the insurance company imposes a $1500 deductible on the homeowners’ insurance.

1. What is the claim distribution for each policy now?
2. Find the expected claim payment and the standard deviation for one policy.
3. Find the expected claim payment and standard deviation for the 100 policies.
4. How does this compare to the insurance without the deductible? What are the advantages and disadvantages of this insurance?
Example: Now suppose the insurance company imposes a maximum claim payment of $125000 in addition to the $1500 deductible on the homeowners’ insurance.

1. What is the claim distribution for each policy now?
2. Find the expected claim payment and the standard deviation for one policy.
3. Find the expected claim payment and standard deviation for the 100 policies.
4. How does this compare to the other types of insurance? What are the advantages and disadvantages of this insurance?

Example: Suppose again the insurance company imposes a $1500 deductible but no benefit limit. Assume that annual inflation is 10%.

1. Over the next five years, what is the claim distribution for each policy?
2. Find the expected claim payment and the standard deviation for one policy over the next five years.
3. Find the expected claim payment and standard deviation for the 100 policies over the next five years.
4. How does inflation affect the expected claim payments and standard deviation of claim payments?
Example: Now suppose the insurance company imposes a maximum claim payment of $125000 in addition to the $1500 deductible on the homeowners’ insurance. Assume that annual inflation is 10%.

1. Over the next five years, what is the claim distribution for each policy?

2. Find the expected claim payment and the standard deviation for one policy over the next five years.

3. Find the expected claim payment and standard deviation for the 100 policies over the next five years.

4. How does inflation affect the expected claim payments and standard deviation of claim payments in this scenario? Why?
Exercises

1. An insurer has a portfolio of 1000 one-year term life insurance policies just issued to 1000 different independent individuals. Each policy will pay $1500 in the event that the policyholder dies within a year. For 500 of the policies, the probability of death is .01 per policyholder and for the other 500 policies, the probability of death is .02 per policyholder. Find the expected value and the standard deviation of the aggregate claim that the insurer will pay.

2. For a one-year dental insurance policy for a family, the model for annual claims is given by

<table>
<thead>
<tr>
<th>Amount of Expense ($X$)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.1</td>
</tr>
<tr>
<td>200</td>
<td>.1</td>
</tr>
<tr>
<td>400</td>
<td>.3</td>
</tr>
<tr>
<td>800</td>
<td>.4</td>
</tr>
<tr>
<td>1500</td>
<td>.1</td>
</tr>
</tbody>
</table>

(a) Find the expected amount and the standard deviation of the family’s claim.

(b) Suppose the insurance company offers this dental insurance to 200 independent families. Find the expected claim amount and standard deviation for the 200 families.

(c) Redo (b), now supposing the insurance company imposes a deductible of $150.

(d) Redo (b), now supposing that the insurance company imposes a benefit limit of $1000 in addition to the deductible of $150.

3. Consider an insurance policy that reimburses annual hospital charges for an insured individual. The probability of any individual being hospitalized in a year is 15%, i.e. $Pr(H = 1) = 0.15$. Once an individual is hospitalized, the charges $X$ have a probability density function

$$f_X(x|H = 1) = 0.1e^{-0.1x} \text{ for } x > 0.$$ 

(a) Find the expected value, standard deviation, and ratio of the standard deviation to the mean (coefficient of variation) of hospital charges for an insured individual.

(b) Determine the expected claim payments, standard deviation, and coefficient of variation for an insurance pool that reimburses hospital charges for 200 individuals. Assume that the claims for each individual are independent of other individuals.