Assignment 7: Due Thursday, October 15

PLEASE SHOW ALL WORK. Each question is worth 5 points.

1. Prove that if $N \leq H \leq G$ and $N \lhd G$, then $N \lhd H$.

2. If $N \leq H$ and $H \leq G$, is $N \lhd G$? Either prove the result or illustrate with a counterexample.

3. Show that if $K$ and $N$ are normal subgroups of $G$, then $K \cap N \lhd G$.

4. Show that if $N \lhd G$ and $H \leq G$, then $H \cap N \lhd H$.

5. Show that if $H \leq Z(G)$, then $H \lhd G$.

6. Show that if $N \lhd G$ and $|N| = 2$, then $N \leq Z(G)$.

7. Show that if $H, K \lhd G$ and $H \cap K = \{1\}$, then $hk = kh$ for all $h \in H$ and $k \in K$.

Other Problems to Consider:
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