1. (a) Let $p$ be a prime number. Fill in the second row of the table to give the number of abelian groups of order $p^n$, up to isomorphism.

<table>
<thead>
<tr>
<th>$n$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Let $p$, $q$, and $r$ be distinct prime numbers. Use the table to find the number of abelian groups, up to isomorphism, of the given order.

i. $p^3q^4r^7$

ii. $(qr)^7$

iii. $q^5r^4q^3$

2. Find all abelian groups, up to isomorphism, of order 192.

3. Give an example of a finite ring $R$ with unity and a subring $S$ of $R$ that is not a ring with unity.

4. Let $R$ be a ring. The center of $R$ is defined by

$$Z(R) = \{ x \in R \mid xy = yx \text{ for all } y \in R \}.$$ 

Show that $Z(R)$ is a subring of $R$.

5. For each of the following, decide if the set under the operations defines a ring. If a ring is formed, state whether the ring is commutative or is a ring with unity.

(a) The set of all pure imaginary complex numbers $ri$ for $r \in \mathbb{R}$ with usual addition and multiplication

(b) $\{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ with usual addition and multiplication

6. Let $R$ be a set with 1 that satisfies every ring axiom except possibly that $a + b = b + a$ for all $a, b \in R$. Show that $R$ is a ring. (Hint: Prove that additive commutativity really does hold in $R$ by computing $(1 + 1)(a + b)$ in two different ways.)

7. Let $R$ be a commutative ring and $a, b \in R$ such that $ab$ is a zero divisor. Prove that either $a$ or $b$ is a zero divisor.

8. Let $R$ be a ring with 1 and no zero divisors. Let $a, b \in R$ satisfy $ab = 1$. Prove that $ba = 1$.

Other Problems to Consider:
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Page 201 #4, 7, 8
Page 208 #2, 4, 12, 24, 31, 33