In Lab 4, we computed cosets in various groups. The collection of cosets forms a new group precisely when the left and the right cosets agree. If the right and the left cosets are the same, then we can define a binary operation \( \# \) on the cosets:

\[
aH \# bH = (ab)H.
\]

The cosets together with this operation form another group.

**Definition:** If \( G \) is a group and \( H \) is a normal subgroup of \( G \), then the collection of cosets is called the **quotient** or **factor group**, and is denoted by \( G/H \).

Recall that \( |G : H| \) is the number of distinct cosets of \( H \) in \( G \). Thus the order of the group \( G/H \) is \( |G : H| \).

1. Consider the group \( S_3 \). Let \( H = \{\rho_0, \rho, \rho^2\} \). In Lab 4, we checked that the left and right cosets agree, so \( H \) is a normal subgroup of \( G \). Therefore we can form the quotient group \( G/H \). Construct a group table for the quotient group \( G/H \). What familiar group has the same group table?
2. Let’s now consider $\mathbb{Z}_{12}$. Set $H = \langle 4 \rangle = \{0, 4, 8\}$. We saw in lab 4 that $H$ is a normal subgroup of $G$. Therefore we can form the quotient group $G/H$. Construct a group table for the quotient group $G/H$. What familiar group has the same group table?