Definition: A **normal distribution** is a continuous, symmetric, bell-shaped distribution of a variable.

The equation for the normal distribution is

$$y = \frac{e^{-(X-\mu)^2/2\sigma^2}}{\sigma \sqrt{2\pi}}.$$ 

Properties of the Normal Distribution:

- A normal distribution curve is bell-shaped.
- The mean, median, mode are all equal and located at the center of the distribution.
- A normal distribution curve is unimodal.
- The curve is symmetric about the mean.
- The curve is continuous; there are no gaps or holes; for each $X$, there is a corresponding value of $y$.
- The curve never crosses the $x$-axis, but gets increasingly closer to the $x$-axis.
- The total area under the normal distribution curve is 1.00
- The area under the part of the normal curve that lies within 1 standard deviation of the mean is 0.68; within 2 standard deviations about 0.95; and within 3 standard deviations about 0.997. (Empirical Rule!!)

Definition: The **standard normal distribution** is a normal distribution with mean 0 and standard deviation 1.

Any normally distributed variable can be transformed into a standard normally distributed variable by:

To find the area under the standard normal curve, draw a picture, shade the area desired, and find the correct figure. Table E on pages 782–783 gives the area to the left of any $z$-value.

1. Finding the area to the left of any $z$-value.
   - Look up the $z$-value in Table E and use the area given.
   - Example: Find the area under the standard normal curve to the left of $z = -1.57$. 
Example: Find the area under the standard normal curve to the left of $z = 1.57$.

2. Finding the area to the right of any $z$-value.
   Look up the $z$-value in Table E and subtract the area from 1.
   Example: Find the area under the standard normal curve to the right of $z = 0.90$.

Example: Find the area under the standard normal curve to the right of $z = -1.34$.

3. Finding the area between any two $z$-values.
   Look up both $z$-values in Table E and subtract the corresponding areas.
   Example: Find the area under the standard normal curve between $z = -2.52$ and $z = 0$.

Example: Find the area under the standard normal curve between $z = 1.57$ and $z = 2.52$.

Example: Find the area under the standard normal curve between $z = -1.52$ and $z = 2.01$.  

Example: Find the area under the standard normal curve between $z = -1.52$ and $z = -0.90$.

**Normal Curve as a Probability Distribution Curve**

In a continuous distribution, the probability of an exact $z$-value is 0 since vertical lines have area 0.

Thus, $P(a \leq z \leq b) = P(a < z < b)$.

Example: Find $P(-1.01 < z < 0.50)$

Example: Find the $z$-value such that the area under the standard normal curve between 0 and the $z$-value is 0.2389.

**Section 6.2: Applications of the Normal Distribution**

Plan of Attack: Transform the original variable to a standard normal distribution variable using the formula and then find the corresponding area with the help of Table E.

Example: Assume that the length of time, $x$, between charges of a cellular phone is normally distributed with a mean of 10 hours and a standard deviation of 1.5 hours. Find the probability that a cell phone will last between 8 and 12 hours between charges.
Example: Suppose the random variable $x$ is the in-city gas mileage for a particular automobile. The probability distribution of $x$ can be approximated by a normal distribution with mean of 27 and standard deviation of 3.

(a) What is the probability that you would purchase an automobile that averages less than 20 miles per gallon for in-city driving?

(b) Suppose you buy one of these models and it does get less than 20 miles per gallon for in-city driving. Should you conclude that your probability model is incorrect?

Sometimes we will be given the percentage and asked to find the data value that corresponds to that percentage. Consider the following example.

Example: Suppose that the scores, $x$, on a college entrance exam are normally distributed with mean of 550 and a standard deviation of 100. A certain prestigious university will only consider for admission those applicants whose scores exceed the 90th percentile. Find the minimum score an applicant must achieve to be considered for admission.
Determining Normality

To assess a distribution for normality, construct a histogram and check its shape. Then one can check skewness using Pearson’s index (PI) of skewness. The formula is

\[ \text{If } PI \geq 1 \text{ or } PI \leq -1, \text{ then the data are significantly skewed.} \]

Finally, check the data for outliers.

Example: Consider the height variable in our questionnaire. Test the variable for normality.

Section 6.3: The Central Limit Theorem

Definition: A **sampling distribution of sample means** is

Definition: **Sampling error** is

Properties of the Distribution of Sample Means

1. 
2. 

Definition: The **standard error of the mean** is

Central Limit Theorem
Note:

1.

2.

Example: Consider the sampling distributions of $\bar{X}$ for different populations and different sample sizes.

Example: A manufacturer of automobile batteries claims that the distributions of life of its best battery has a mean of 54 months and a standard deviation of 6 months. Suppose a consumer group decides to check this claim by purchasing a sample of 50 of these batteries and subjecting them to tests that determine battery life.

(a) Assuming that the manufacturer’s claim is true, describe the sampling distribution of the mean lifetime of a sample of 50 batteries.

(b) Assuming that the manufacturer’s claim is true, what is the probability that the consumer group’s sample has a mean life of 52 or fewer months?
Example: The distribution of the number of barrels of oil produced by a certain oil well each day for the past 3 years has a mean of 400 and a standard deviation of 75.

(a) Describe the sampling distribution of the mean number of barrels produced per day for samples of 40 production days drawn from the last 3 years.
(b) What is the approximate probability that the sample mean will be greater than 425?
(c) What is the approximate probability that the sample mean will be less than 400?

When do we use
\[ z = \frac{X - \mu}{\sigma/\sqrt{n}} \]
instead of
\[ z = \frac{X - \mu}{\sigma} \]?

Example: The average number of pounds of meat that a person consumes a year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.

(a) Find the probability that a person selected at random consumes less than 224 pounds per year.
(b) If a sample of 40 individuals is selected, find the probability that the mean of the sample will be less than 224 pounds per year.